

Problem 1.49

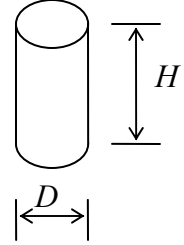
[Difficulty: 4]

1.49 Using the nominal dimensions of the soda can given in Problem 1.47, determine the precision with which the diameter and height must be measured to estimate the volume of the can within an uncertainty of ± 0.5 percent.

Given: Dimensions of soda can: $D = 66 \text{ mm}$, $H = 110 \text{ mm}$

Find: Measurement precision needed to allow volume to be estimated with an uncertainty of ± 0.5 percent or less.

Solution: Use the methods of Appendix F:



Computing equations:

$$\mathcal{V} = \frac{\pi D^2 H}{4}$$

$$u_{\mathcal{V}} = \pm \left[\left(\frac{H}{\mathcal{V}} \frac{\partial \mathcal{V}}{\partial H} u_H \right)^2 + \left(\frac{D}{\mathcal{V}} \frac{\partial \mathcal{V}}{\partial D} u_D \right)^2 \right]^{\frac{1}{2}}$$

Since $\mathcal{V} = \frac{\pi D^2 H}{4}$, then $\frac{\partial \mathcal{V}}{\partial H} = \frac{\pi D^2}{4}$ and $\frac{\partial \mathcal{V}}{\partial D} = \frac{\pi D H}{2}$. Letting $u_D = \pm \frac{\delta x}{D}$ and $u_H = \pm \frac{\delta x}{H}$, and substituting,

$$u_{\mathcal{V}} = \pm \left[\left(\frac{4H}{\pi D^2 H} \frac{\pi D^2}{4} \frac{\delta x}{H} \right)^2 + \left(\frac{4D}{\pi D^2 H} \frac{\pi D H}{2} \frac{\delta x}{D} \right)^2 \right]^{\frac{1}{2}} = \pm \left[\left(\frac{\delta x}{H} \right)^2 + \left(\frac{2\delta x}{D} \right)^2 \right]^{\frac{1}{2}}$$

Solving,

$$u_{\mathcal{V}}^2 = \left(\frac{\delta x}{H} \right)^2 + \left(\frac{2\delta x}{D} \right)^2 = (\delta x)^2 \left[\left(\frac{1}{H} \right)^2 + \left(\frac{2}{D} \right)^2 \right]$$

$$\delta x = \pm \frac{u_{\mathcal{V}}}{\left[\left(\frac{1}{H} \right)^2 + \left(\frac{2}{D} \right)^2 \right]^{\frac{1}{2}}} = \pm \frac{0.005}{\left[\left(\frac{1}{110 \text{ mm}} \right)^2 + \left(\frac{2}{66 \text{ mm}} \right)^2 \right]^{\frac{1}{2}}} = \pm 0.158 \text{ mm}$$

Check:

$$u_H = \pm \frac{\delta x}{H} = \pm \frac{0.158 \text{ mm}}{110 \text{ mm}} = \pm 1.44 \times 10^{-3}$$

$$u_D = \pm \frac{\delta x}{D} = \pm \frac{0.158 \text{ mm}}{66 \text{ mm}} = \pm 2.39 \times 10^{-3}$$

$$u_{\mathcal{V}} = \pm [(u_H)^2 + (2u_D)^2]^{\frac{1}{2}} = \pm [(0.00144)^2 + (0.00478)^2]^{\frac{1}{2}} = \pm 0.00499$$

If δx represents half the least count, a minimum resolution of about $2 \delta x \approx 0.32 \text{ mm}$ is needed.